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Optical Arbitrary Waveform Generators Based on Temporal and Spectral Shaping of Optical Pulses in Nonlinear Metamaterials

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14. ABSTRACT The final goal of the research project is the theoretical design of new optical devices on a Lithium Niobate and Tantalate platform; in particular we will focus on low-cost integrated optical pulse shapers on periodically poled Lithium Niobate and Tantalate waveguides. Our approach will be mainly used to design pulse shapers, but it also might open new design perspectives for other optical devices. In fact the continuous progress in the control of linear and nonlinear properties of optical waveguides has led nowadays to the possibility of engineering the design of integrated photonic components down to the sub micrometer space scale. Such waveguides exhibit a spatially periodic micro or nano-structure which enables the engineering of their dispersive and nonlinear properties to a degree that was previously unconceivable.. This key technology has opened up new possibilities in diverse applications; in particular, from the system point of view, this entails that we can now imagine devices with performances that were totally unconceivable only a few years ago. As research continues to advance in this area new frontiers are explored: currently there is intense activity directed, on the one hand, to achieve even higher precision in the control of the linear and nonlinear properties of optical devices, and on the other hand to conceive new device functionalities, thanks to the increased number of degrees of freedom of the fabrication parameters. The latter issue might also open a new design perspective: up to now optical devices have been mainly designed on a try and test procedure that was reasonable since only a small number of parameters was free for the design (i.e. controllable in the fabrication process). With the new incredible number of degrees of freedom that are nowadays available, this old style design might hide the possibility of the realization of new device functionalities: one of the main theoretical goals of our project is thus to introduce and test new theoretical and numerical techniques to open the way for new design procedures for optical waveguide components (for example genetic algorithms and iterative procedures based on optimal control theory used as design tools.					
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Optical Arbitrary Waveform Generators Based on Temporal and Spectral Shaping of Optical Pulses in Nonlinear Metamaterials

Comprehensive final report

ABSTRACT.

During the project, using an approach based on optimal control theory, we have first implemented a pulse shaper design tool. Using as tuning parameter the longitudinal dependence of the second order response $\chi^{(2)}$ of the medium where propagation takes place, the goal has been achieved by minimizing the distance between a given target pulse and the second harmonic at output of a device for second harmonic generation. The outcome of this first theoretical demonstration of pulse shaping technique is the function $\chi^{(2)}(z)$ which is necessary to obtain the desired shaping effect. Starting from results we have previously obtained [1], we have demonstrated here both wavelength tunability and arbitrary pulse shaping capability.

In this framework we have also explored ultrabroadband optical pulse propagation in nonlinear quadratic media [2, 3] to address engineered supercontinuum generation as a valuable tool for spectral shaping.

According to the work plan described in the research proposal, during the third month of the project activities, Matteo Conforti and Costantino De Angelis have been visiting their research partners in the United States, namely:

- Prof. Alejandro B. Aceves and Prof. Ildar Gabitov at the Department of Mathematics of the Southern Methodist University, in Dallas.
- Dr. Michael Scalora, AMSRD-WSS Charles M. Bowden Research Facility at Redstone Arsenal, Huntsville.
- Prof. Triantaphyllos R. Akylas, Department of Mechanical Engineering, Massachusetts Institute of Technology, Cambridge, Massachusetts.

During the seventh month of the project Stefan Wabnitz has been visiting Prof. Alejandro B. Aceves at the Department of Mathematics of the Southern Methodist University, in Dallas.

The research work carried on during these visits has been very fruitful and has been a real boost for the project; in particular we have identified there the need for a better understanding of plasmonic wave dynamics, thereby providing the theoretical underpinnings of potential innovative applications. To this end, a combined analytical /numerical study of light propagation in various structures composed of metals and dielectrics has been pursued.

In this framework periodic structures, consisting of metal and dielectrics have been investigated and we have demonstrated the existence of new solitary wave solutions of a model describing light propagation in nonuniform (linearly and nonlinearly) waveguide arrays. This general model describes energy localization and transport in different physical settings, ranging from metal-dielectric (i.e. plasmonic) to photonic crystal waveguides. The solitons exist for both focusing, defocusing and even for alternating focusing-defocusing nonlinearity [4].

IN DETAIL.

In the field of modelling plasmonic nanostructures and engineering of their guiding properties, the work done in this project has paved the way to the management of broadband light propagation. Moreover, all the numerical tools already developed by UNIBS to analyse the modal properties of metal-dielectric structures and the field evolution in the linear regime are to be considered as one of the result of this research project. As far as diffraction management is concerned, we have considered propagation in a structure where two different plasmonic devices are involved. The first one is a system composed of alternating metal (30 nm) and dielectric (120 nm) layers.

In figure 1 we show the dispersion relation and the profiles of the two modes supported by this directional coupler (to be called DC1). In stark contrast with conventional waveguides, the fundamental mode of this structure is odd. Moreover we can see that the fundamental mode has one node whereas the second one has no nodes.

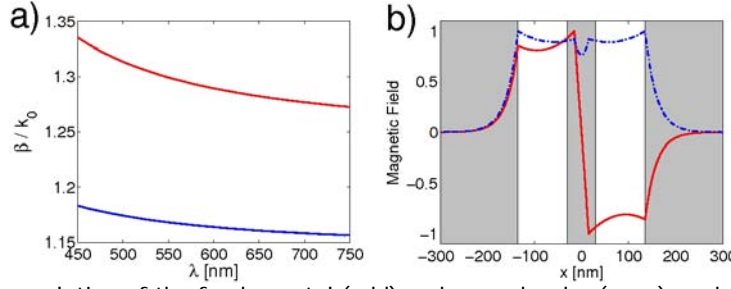


Figure 1: a) Dispersion relation of the fundamental (odd) and second order (even) mode of the coupler DC1. b) Fundamental (red) and second order (blue) mode @ 600nm.

A second well known plasmonic guiding structure is the metal nanoparticle array, where the energy transport is supplied by electromagnetic resonant coupling between metal particles arranged in a linear chain. We have studied double nanoparticle chains, where the electromagnetic energy is confined between two linear chains. As an example we considered double chain waveguides composed of silver nanoparticles with a radius of 50 nm in a dielectric host with a longitudinal separation of 110 nm and a distance between the chains of 150 nm. In figure 2 we show the dispersion relation and the mode profiles of two coupled waveguides composed of three nanoparticle chains (to be called directional coupler DC2). As opposed to the previous directional coupler, the fundamental mode here is even and the second order mode is odd.

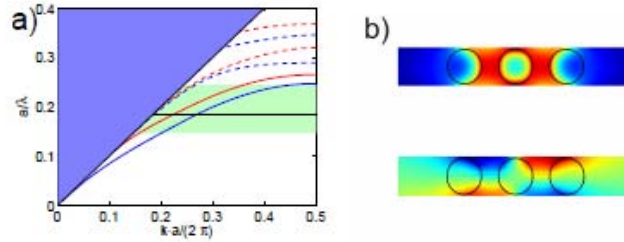


Figure 2: a) Dispersion relation of the coupler DC2. Thick curves, fundamental (even) and second order (odd) mode. Dashed curves, higher order modes. Thin horizontal line denotes a wavelength of 600 nm. b) Fundamental and second order mode @ 600nm.

We have then considered the uniform array obtained using directional couplers DC1 (DC2) as basic building blocks. The field evolution along the waveguide arrays has been simulated without any approximation by solving Maxwell's equations through a frequency-domain finite-element method, using arrays composed by 17 waveguides. The central waveguide of the array is excited with a Gaussian field, which spreads during propagation and generates the typical diffraction pattern observed also in conventional waveguide arrays: two outermost wings and a few less intense peaks in the central waveguides. The same qualitative behavior is observed for both arrays since the intensity evolution is not influenced by the diffraction sign. On the other hand the phase front curvature of the propagating field depends on the diffraction sign. Therefore, if we alternate arrays characterized by normal and anomalous diffraction, the input field shape can be periodically recovered as demonstrated in figure 3.

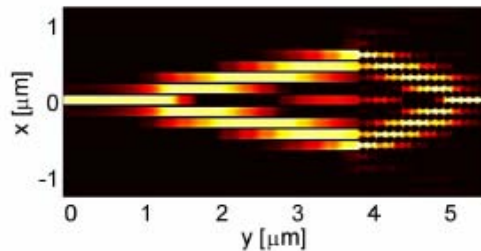


Figure 3: Time average power flow along propagation in the diffraction managed device @ 600nm.

In the nonlinear regime, we have obtained solitary wave solutions of a model describing light propagation in binary (linearly and nonlinearly) waveguide arrays. This model describes energy localization and transport in

various physical settings, ranging from metal--dielectric (i.e. plasmonic) to photonic crystal waveguides. The solitons exist for focusing, defocusing and even for alternating focusing--defocusing nonlinearity.

In this project we have considered a binary array designed in such a way that the coupling between successive waveguides switches periodically from C to $-C(1+\epsilon)$, thus opening a gap centered at zero Bloch momentum in the linear dispersion relation. We consider also a binary Kerr nonlinearity and we look for self-sustained nonlinear propagation in the form of gap solitons in such a structure. Specifically, extending previously derived results, we obtain in the continuum limit exact analytical solutions for both stationary and "walking" gap solitons moving along the spatial coordinate with a tunable velocity.

We have also focused our attention on the existence domain for bright gap solitons. These solitons belong to a family with two free parameters: the velocity v and the energy unbalance.

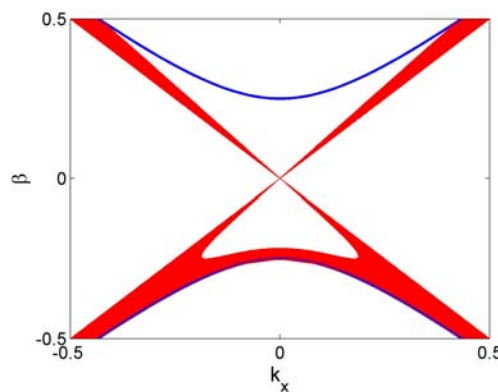


Figure 4: Existence conditions for gap soliton solutions on transverse and longitudinal wave number. Continuous blue line refers to the dispersion relation of the linear problem; the red region is the existence domain for gap soliton solutions.

Such solitons display several interesting and unusual features, unique to this type of waveguide structures, and are possible even in the case of alternating focusing-defocusing nonlinearity [4].

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Energy localization and transport in binary waveguide arrays

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We obtain solitary-wave solutions of a model describing light propagation in binary (linearly and nonlinearly) waveguide arrays. This model describes energy localization and transport in various physical settings, ranging from metal-dielectric (i.e., plasmonic) to photonic crystal waveguides. The solitons exist for focusing, defocusing, and even for alternating focusing-defocusing nonlinearity.

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I. INTRODUCTION

Discrete optics in coupled waveguides has been an area of intense research activities during the last three decades (see [1] for a recent review). Most efforts have been devoted to the analysis of linear and nonlinear properties of uniform waveguide arrays, i.e., arrays composed of equally spaced identical waveguides, and both one-dimensional and multidimensional configurations have been considered theoretically as well as experimentally [2–8].

On the other hand, nonuniform waveguide arrays offer a richer setting where engineering of the periodic structure can provide further degrees of freedom. In this context, zigzag waveguide arrays (i.e., the cascade of arrays characterized by alternating tilt angles) have been introduced to get diffraction management [9]. Binary arrays composed of waveguides with different wave numbers have been thoroughly studied (see [10–13] and related works) since they exhibit interesting features, such as double refraction, due to their intrinsic two-band nature. Moreover, binary arrays with different coupling coefficients have been considered, since they might offer a more feasible experimental framework in which to exploit a two-band structure in the linear and nonlinear regimes [14,15]. In this instance, the use of photonic crystal waveguides [16] or waveguides based on plasmonic confinement [17,18] offers a unique setting in which to exploit propagation in the so-called alternating positive and negative coupling regime [19,20]. Efremidis *et al.* [20], in particular, studied nonlinearly uniform arrays where the coupling coefficients are equal in modulus but of opposite sign. In this case the structure does not possess a gap in the linear spectrum, and can be reduced to a uniform array by a phase transformation. Exploiting this transformation, families of discrete solitons were calculated starting from the well-known discrete Schrödinger equation.

In this paper we consider a binary array designed in such a way that the coupling between successive waveguides switches periodically from C to $-C(1 + \epsilon)$, thus opening a gap centered at zero Bloch momentum in the linear dispersion relation. We consider also a binary Kerr nonlinearity and we look for self-sustained nonlinear propagation in the form of gap solitons in such a structure. Specifically, extending previously derived results [21–30], we obtain in the continuum limit exact analytical solutions for both stationary and “walking” gap solitons moving along the spatial coordinate with a tunable velocity.

II. THEORETICAL MODEL AND SOLITON SOLUTIONS

According to coupled mode theory and taking into account third-order nonlinearities in the form of a pure Kerr effect, the governing equations read as [10]

$$i E'_{nz} + \beta_n E'_n + C_{n-1} E'_{n-1} + C_{n+1} E'_{n+1} + \chi_n |E'_n|^2 E'_n = 0,$$

where E'_n is the amplitude of the modal field $M_n(x)$ of the n th waveguide; β_n is the propagation constant of each individual waveguide ($\beta_n = \beta + \Delta\beta/2$ for n even and $\beta_n = \beta - \Delta\beta/2$ for n odd); χ_n , the site-dependent nonlinear coefficient, is γ_1 (γ_2) for n even (odd); and C_{n-1}, C_{n+1} are the coupling coefficients with the $(n-1)$ th and the $(n+1)$ th waveguides, respectively. In the specific case of interest, $C_{n-1} = C_1$ and $C_{n+1} = C_2$ when n is even, whereas $C_{n-1} = C_2$ and $C_{n+1} = C_1$ when n is odd. We then perform the transformation $E'_n = E_n \exp(i\beta z)$ and we separately consider the mode amplitudes in the even and odd waveguides. Finally, $E_{2n} = A_n$ and $E_{2n-1} = B_n$ are governed by the following two sets of coupled equations with constant coefficients:

$$\begin{aligned} i A_{nz} + \frac{\Delta\beta}{2} A_n + C_1 B_n + B_{n+1} + \gamma_1 |A_n|^2 A_n &= 0, \\ i B_{nz} - \frac{\Delta\beta}{2} B_n + A_{n-1} + C_1 A_n + \gamma_2 |B_n|^2 B_n &= 0, \end{aligned} \quad (1)$$

where C_2 has been set equal to 1, without loss of generality.

Assuming Bloch-wave disturbances, $(A_n, B_n) \propto \exp[i(nk_x + k_z z)]$, the linear dispersion relation of Eqs. (1) reads

$$k_z^2 = \left(\frac{\Delta\beta}{2}\right)^2 + C_1^2 + 1 + 2C_1 \cos k_x.$$

Note that a band gap opens whenever $\Delta\beta \neq 0$ and/or for $C_1 \neq \pm 1$, the band edges corresponding to the wave number $k_x = 0$ for $C_1 < 0$ and $k_x = \pi$ for $C_1 > 0$. Moreover, there is numerical evidence [10,28] that discrete solitons can reside inside this gap. We shall make a comprehensive analytical study of stationary and moving gap solitons on the basis of an equivalent continuous model.

Specifically, for $C_1 < 0$, in the neighborhood of $k_x = 0$, we use the expansions

$$\begin{aligned} A_{n\pm 1}(z) &= u(x, z) \pm u_x(x, z) + \frac{1}{2} u_{xx}(x, z) + \dots, \\ B_{n\pm 1}(z) &= w(x, z) \pm w_x(x, z) + \frac{1}{2} w_{xx}(x, z) + \dots \end{aligned}$$

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to obtain (as a first-order approximation)

$$\begin{aligned} i u_z + \frac{\Delta\beta}{2} u + w_x + \epsilon w + \gamma_1 |u|^2 u &= 0, \\ i w_z - \frac{\Delta\beta}{2} w - u_x + \epsilon u + \gamma_2 |w|^2 w &= 0, \end{aligned} \quad (2)$$

where $C_1 = -1 + \epsilon$. This equation system also arises in the neighborhood of $k_x = \pi$ for $C_1 = 1 + \epsilon$, following a similar expansion procedure after the change of variables $(A_n, B_n) \rightarrow (-1)^n (A_n, B_n)$.

We now look for both stationary and walking self-confined solutions of the system defined by Eqs. (2). To this end, we use the following trial functions [23]:

$$\begin{aligned} u(x, z) &= \frac{1}{2} [K_1 g_1(\xi) + i K_2 g_2(\xi)] \exp(i\psi \cos Q), \\ w(x, z) &= \frac{1}{2i} [K_1 g_1(\xi) - i K_2 g_2(\xi)] \exp(i\psi \cos Q), \\ \xi &= \frac{x + vz}{\sqrt{1-v^2}}, \quad \psi = \frac{vx + z}{\sqrt{1-v^2}}, \\ K_1 &= \left(\frac{1+v}{1-v} \right)^{1/4}, \quad K_2 = \left(\frac{1-v}{1+v} \right)^{1/4} \end{aligned} \quad (3)$$

with $g_{1,2}$ two arbitrary complex functions, $-1 \leq v \leq 1$ and $0 \leq Q \leq \pi$. Although not necessary, for the sake of simplicity, from now on we set $\Delta\beta = 0$ (i.e., the biatomic nature of the array is left to the coupling coefficients only).

Substitution of the ansatz (3) into Eqs. (2) gives ($s = \gamma_1 + \gamma_2$, $d = \gamma_1 - \gamma_2$)

$$\begin{aligned} -\dot{g}_1 + i \cos(Q) g_1 + i \epsilon g_2 + \frac{s}{8i} (K_1^4 |g_1|^2 g_1 + 2 |g_2|^2 g_1 - g_2^2 g_1^*) \\ - \frac{d}{8} (-K_2^2 |g_2|^2 g_2 - 2 K_1^2 |g_1|^2 g_2 + K_1^2 g_1^2 g_2^*) &= 0, \\ \dot{g}_2 + i \cos(Q) g_2 + i \epsilon g_1 + \frac{s}{8i} (K_2^4 |g_2|^2 g_2 + 2 |g_1|^2 g_2 - g_1^2 g_2^*) \\ - \frac{d}{8} (K_1^2 |g_1|^2 g_1 + 2 K_2^2 |g_2|^2 g_1 - K_2^2 g_2^2 g_1^*) &= 0. \end{aligned}$$

These equations have the invariant $P = |g_1|^2 - |g_2|^2$; as we are interested in bright solitons, we set $P = 0$, so that $|g_1|^2 = |g_2|^2$ and $g_{1,2}(\xi) = f(\xi) \exp[i\theta_{1,2}(\xi)]$. Finally, using $\eta = f^2$ and $\mu = \theta_1 - \theta_2$, we get

$$\begin{aligned} \dot{\eta} &= -\frac{\partial H}{\partial \mu}, \quad \dot{\mu} = \frac{\partial H}{\partial \eta}, \\ H &= 2\eta(\epsilon \cos \mu + \cos Q) \\ &\quad - \frac{s}{8} \eta^2 \left(\frac{K_1^4}{2} + \frac{K_2^4}{2} + 2 - \cos(2\mu) \right) \\ &\quad - \frac{d}{4} \eta^2 (K_1^2 + K_2^2) \sin \mu. \end{aligned} \quad (4)$$

Equations (4) represent a one-dimensional (thus integrable) Hamiltonian system, and solitary-wave solutions correspond to the separatrix trajectories that are homoclinic to (i.e., emanate from and return to) the unstable fixed points of (4). In the following we assume $s > 0$, since the results can be easily extended to the case $s < 0$ by the substitution $\mu \rightarrow \mu + \pi$, $Q \rightarrow \pi - Q$.

Bright solitons emanate from the unstable fixed point $(\eta_0, \mu_0) = (0, \pm \arccos[-\cos(Q)/\epsilon])$ and correspond to level curves of the Hamiltonian $H(\eta_0, \mu_0) = 0$. By exploiting $H = 0$, we can derive the expression of η as a function of μ from the definition of H :

$$\eta = \frac{16(\epsilon \cos \mu + \cos Q)}{s \left[\frac{K_1^4}{2} + \frac{K_2^4}{2} + 2 - \cos(2\mu) \right] + 2d(K_1^2 + K_2^2) \sin \mu}. \quad (5)$$

By inserting Eq. (5) into $\dot{\mu} = \frac{\partial H}{\partial \eta}$, it follows that

$$\dot{\mu} = -2(\cos Q + \epsilon \cos \mu). \quad (6)$$

This equation can be easily integrated to obtain

$$\mu(\xi) = -2 \arctan \left[\sqrt{\frac{\epsilon + \cos Q}{\epsilon - \cos Q}} \tanh[\sqrt{\epsilon^2 - \cos(Q)^2} \xi] \right]. \quad (7)$$

and, upon substitution in Eq. (5),

$$\eta(\xi) = \frac{\text{sech}(\delta \xi)^2 (\cos Q + \epsilon)}{-4db_1 \alpha \tanh(\delta \xi) + sb_2 [1 + \alpha^2 \tanh(\delta \xi)^2] - 2s \frac{[1 - \alpha^2 \tanh(\delta \xi)^2]^2}{1 + \alpha^2 \tanh(\delta \xi)^2}}, \quad (8)$$

where $\alpha = \sqrt{\frac{\epsilon + \cos(Q)}{\epsilon - \cos(Q)}}$, $\delta = \sqrt{\epsilon^2 - \cos(Q)^2}$, $b_1 = K_1^2 + K_2^2$, $b_2 = \frac{K_1^4}{2} + \frac{K_2^4}{2} + 3$.

Once we get the solutions $\eta(\xi)$ and $\mu(\xi)$, we can find the fields $u(x, z)$ and $w(x, z)$ by substitution in (3) and solving

$$\begin{aligned} \dot{\theta}_1 &= \cos Q + \epsilon \cos \mu - \frac{s}{8} \eta [K_1^4 + 2 - \cos(2\mu)] \\ &\quad - \frac{d}{8} \eta (3K_1^2 + K_2^2) \sin \mu. \end{aligned} \quad (9)$$

The expression of θ_1 is rather cumbersome except for stationary solutions ($v = 0$). In fact by inserting (5) and (6) in (9) it is straightforward to show that $\theta_1 = \mu/2$.

We now focus our attention on the existence domain for bright gap solitons. These solitons belong to a family with two free parameters: the velocity v ($-1 \leq v \leq 1$) and Q ($\arccos \epsilon \leq Q \leq \pi - \arccos \epsilon$). From Eq. (5) we note, however, that, as the array parameters (ϵ, s, d) are changed, the amplitude η can diverge at some points, entailing that some (v, Q) couples are not allowed.

More specifically, for a given Q , we find that $\eta(\xi)$ is bounded for all ξ , and bright soliton solutions can exist only above a critical velocity v ; as long as $s > |d|$ this critical velocity is 0 so that all possible v and Q values in the (Q, v) plane can be attained; however, when $|d| \geq s$ a different situation arises: for $\arccos \epsilon \leq Q \leq \pi/2$, solutions only exist above a critical velocity v_{cr} :

$$|v_{cr}| = \frac{\sqrt{2}}{s} \sqrt{s^2 - d^2 + |d| \sqrt{d^2 - s^2}}. \quad (10)$$

For $\pi/2 \leq Q \leq \pi - \arccos \epsilon$ such a critical velocity does not exist; however, bright soliton solutions are permitted only for $Q \geq \pi - \arccos(\epsilon \sqrt{t})$ with t given by

$$t = \frac{s^2(3 + f_1) - (df_2)^2 + \sqrt{(df_2)^4 - 2s^2d^2f_2^2(1 + f_1)}}{2s^2}, \quad (11)$$

$$f_1 = \frac{K_1^4 + K_2^4}{2}, \quad f_2 = K_1^2 + K_2^2.$$

This last condition can be derived by looking at the phase plane (η, μ) . When $Q = \pi - \arccos(\epsilon \sqrt{t})$, another unstable fixed point exists characterized by $\mu_p = -\text{sgn}(d) \arccos[-\cos(Q)/\epsilon]$, $\eta_p = 8\epsilon^2 \sqrt{1 - \cos(Q)^2/\epsilon^2} \sec(Q)/[-|d|(K_1^2 + K_2^2) + 2s\sqrt{1 - \cos(Q)^2/\epsilon^2}]$ and having Hamiltonian $H(\eta_p, \mu_p) = 0$. In this instance the separatrix trajectory is heteroclinic, connecting the points (η_0, μ_0) and (η_p, μ_p) , and the resulting solution corresponds to a kink soliton. If we increase Q above this threshold, the trajectories in the phase plane become unbounded, preventing the existence of localized solutions.

The existence conditions on v and Q can be easily translated into conditions on the soliton transverse phase $k_x = v \cos(Q)/(\sqrt{1 - v^2})$ and propagation constant $k_z = \cos(Q)/(\sqrt{1 - v^2})$, as illustrated in Fig. 1.

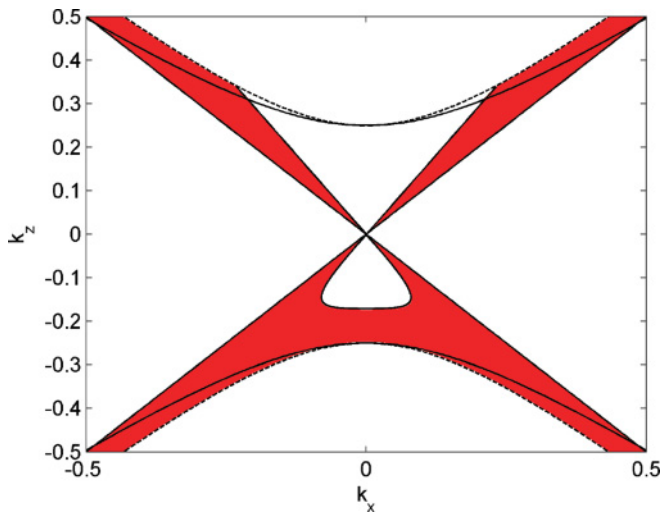


FIG. 1. (Color online) Existence conditions on k_x and k_z . Continuous line refers to the dispersion relation of the discrete problem; dashed line shows the dispersion relation of the continuous approximation and the filled region corresponds to the existence domain of the gap soliton solutions ($\epsilon = 0.25$, $s = 2$, $d = 2.1$).

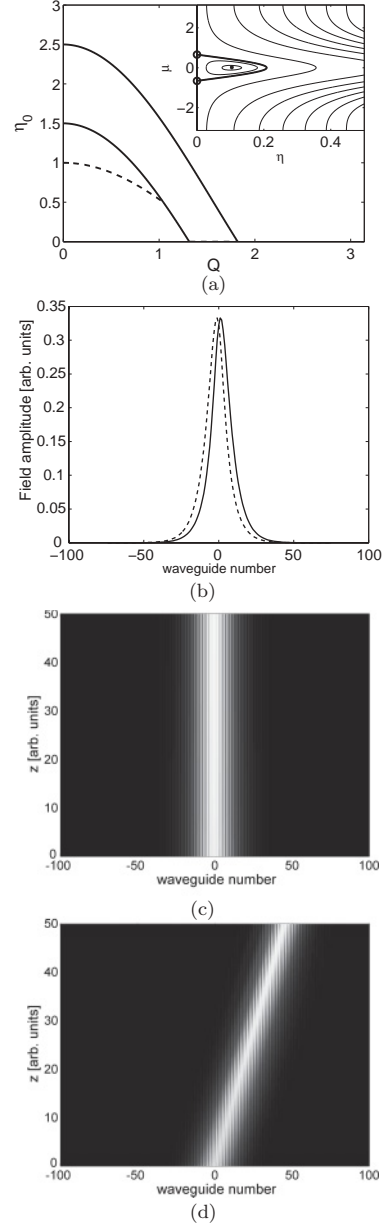


FIG. 2. (a) Bifurcation diagram of the Hamiltonian system ($v = 0$): continuous line for stable centers, dashed line for unstable saddles; inset shows phase plane for $Q = 1.77$ (open circle indicates saddle, filled circle indicates stable center); (b) field amplitude in even (continuous) and odd (dashed) waveguides ($v = 0$); (c) field evolution along the array for $v = 0$; (d) field evolution along the array for $v = 0.5$. In all panels, $\epsilon = 0.25$, $Q = 1.77$, $s = 2$, $d = 0$.

III. EXAMPLES

In this section, we discuss some specific examples of the soliton solutions derived earlier. We also show the robustness of our solutions in some representative cases where we consider propagation in different arrays.

As a first example, we consider an array with all the waveguides having the same nonlinear response ($s = 2, d = 0$). In this case, bright soliton solutions do exist for $\arccos \epsilon \leq Q \leq \pi - \arccos \epsilon$ as one can also infer from the bifurcation diagram of Fig. 2(a), that shows the amplitude η_0 of the

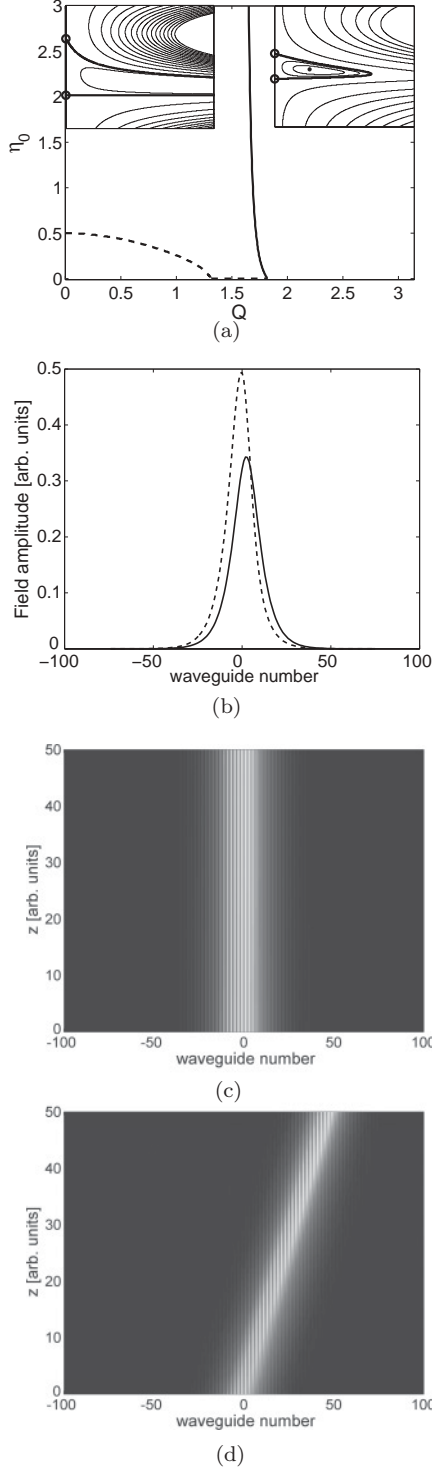


FIG. 3. (a) Bifurcation diagram of the Hamiltonian system ($v = 0$): continuous line for stable centers, dashed line for unstable saddles; left (right) inset shows phase plane for $Q = 1.5$ ($Q = 1.77$) (open circle indicates saddle, filled circle indicates stable center); (b) field amplitude in even (continuous) and odd (dashed) waveguides ($v = 0$); (c) field evolution along the array for $v = 0$; (d) field evolution along the array for $v = 0.5$. In all panels, $\epsilon = 0.25$, $Q = 1.77$, $s = 2$, $d = 2$.

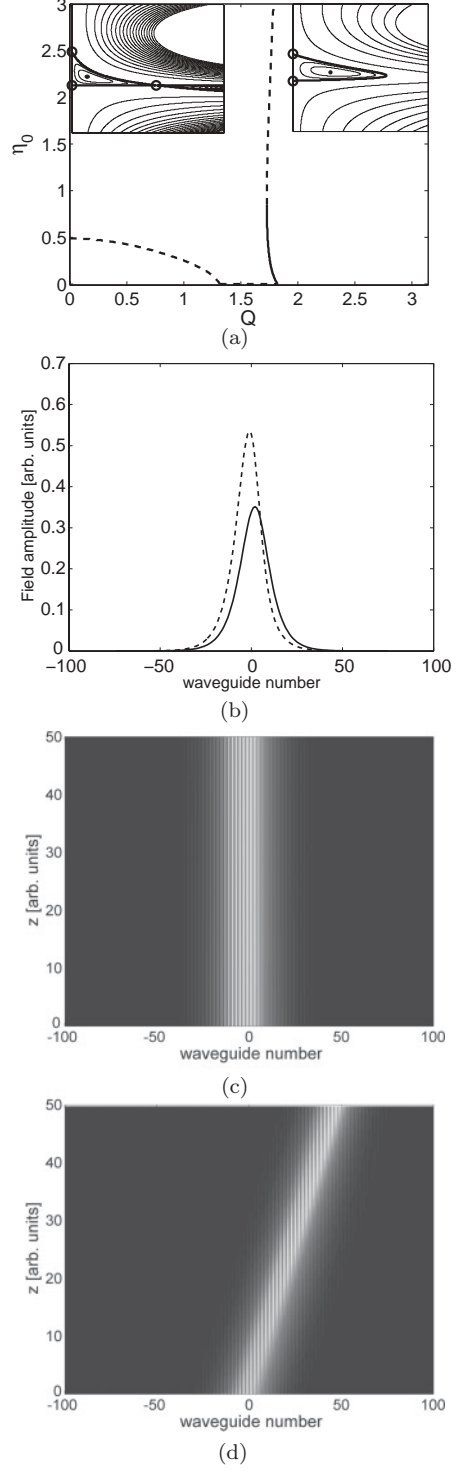


FIG. 4. (a) Bifurcation diagram of the Hamiltonian system ($v = 0$): continuous line for stable centers, dashed line for unstable saddles; left (right) inset shows phase plane for kink (bright) soliton (open circle indicates saddle, filled circle indicates stable center); (b) field amplitude in even (continuous) and odd (dashed) waveguides ($v = 0$); (c) field evolution along the array for $v = 0$; (d) field evolution along the array for $v = 0.5$. In all panels, $\epsilon = 0.25$, $Q = 1.77$, $s = 2$, $d = 2.1$.

fixed points of system (4) as a function of the parameter Q . In fact, in this interval both the unstable saddle $\eta_0 = 0$ and a stable center with $\eta_0 \neq 0$ exist. From the phase plane depicted in the inset (corresponding to $Q = 1.77$), it is evident that the separatrix describing the soliton emanates from $(\eta_0, \mu_0) = (0, \arccos[-\cos(Q)/\epsilon])$, turns around the center and returns to the fixed point with vanishing η . Note that in this situation ($d = 0$) the $v = 0$ case shows perfect mirror symmetry between the field in the even and odd sites [see Fig. 2(b)]; as d increases this mirror symmetry is obviously lost. Figures 2(c) and 2(d) show the propagation of a stationary ($v = 0$) and a moving ($v = 0.5$) soliton.

The second example we are considering in this section corresponds to $s = 2, d = 2$ (i.e., an interlaced linear-nonlinear array). As one can see from Fig. 3(a), for this choice of parameters bright soliton solutions for $v = 0$ exist only for $\pi/2 < Q \leq \pi - \arccos \epsilon$, because the stable center does not exist for $Q < \pi/2$. In this instance, the phase portrait is qualitatively different for Q greater or less than $\pi/2$ [right and left insets of Fig. 3(a)]. For $Q > \pi/2$ the phase portrait is similar to the $d = 0$ case, except for the asymmetry with respect to μ . For $Q < \pi/2$ the separatrix emanating from the saddle is not closed and separates orbits of unbounded motion from periodic motion. As a consequence solitons do not exist.

It is remarkable to note that even in the case of interlaced focusing-defocusing nonlinearities soliton solutions still exist as clearly demonstrated in Fig. 4 for $s = 2, d = 2.1$; this applies also to solutions walking along the array as shown in Fig. 4(d). Note also that, as can be seen from Fig. 4(a), in this case we do not have bright soliton solutions for $Q < 1.73$; however, as we have already noted above, in the presence of a nonzero transverse velocity we have access to this region of Q values. This is what we can see in Fig. 5 where propagation in an interlaced focusing-defocusing array is shown for $Q = 1.72$ and $v = 0.5$; note that, remarkably, this last case corresponds to a situation where we do not have bright soliton solutions with zero transverse velocity.

Another interesting feature of the interlaced focusing-defocusing case is the existence of flat-top and kink solitons, due to the presence of an additional saddle in the bifurcation diagram. It is possible that the two saddles possess the same

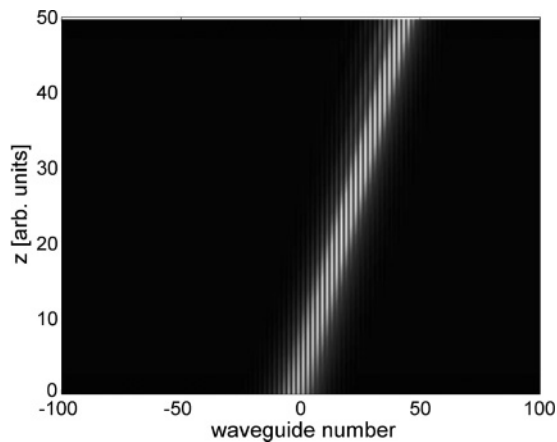
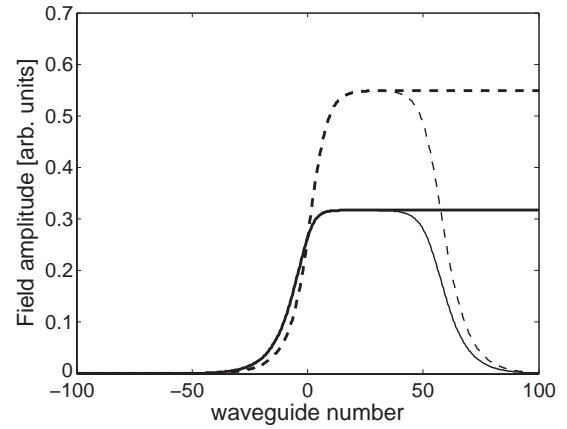
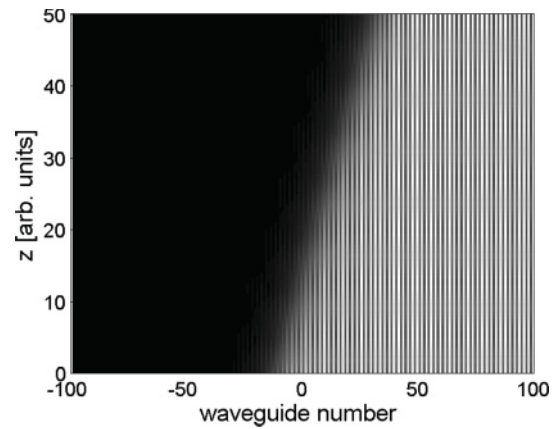


FIG. 5. Field evolution along the array for $v = 0.5$; here $\epsilon = 0.25$, $Q = 1.72$, $s = 2$, $d = 2.1$.



(a)



(b)

FIG. 6. (a) Example of a flat-top (thin lines) and a kink (thick lines) soliton. Here $\epsilon = 0.25$, $Q = 1.786\,553\,604\,650\,208$ for the dark soliton ($Q = 1.786\,553\,7$ for the flat-top soliton), $s = 2$, $d = 2.5$, $v = 0.5$. (b) Field evolution along the array for the kink soliton.

Hamiltonian: in this case the heteroclinic orbit connecting the two points gives rise to a kink soliton [left inset of Fig. 4(a)]. As Q approaches the existence limit defined by (11), bright solitons become wider and eventually take a kinklike shape. An example of this kind of solution is reported in Fig. 6(a). Figure 6(b) shows the propagation of the kink soliton with velocity $v = 0.5$.

IV. CONCLUSIONS

We have analyzed a model describing light propagation in a binary array, accounting for alternating positive and negative linear coupling as well as nonuniform nonlinearity. This model can be applied in different physical settings such as plasmonic, Bragg, and photonic crystal waveguides. We derived exact bright and kink soliton solutions in the long-wavelength (i.e., continuous) limit. Such solitons display several interesting and unusual features, unique to this type of waveguide structure, and are possible even in the case of alternating focusing-defocusing nonlinearity.

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